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School Psychologists as Consumers of Research: What School Psychologists Need to Know About Factor Analysis

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School Psychologists as Consumers of Research: What School Psychologists Need to Know About Factor Analysis

Factor analysis is a versatile class of psychometric techniques used by researchers to provide insight into the psychological dimensions (factors) that may account for the relationships among variables in a given dataset. The primary goal of a factor analysis is to determine a more parsimonious set of variables (i.e., fewer than the number of original variables) to generate a model for the data that can be used to aid the interpretation of those constructs. However, the literature on factor analysis can be mathematically complex and filled with contradictory viewpoints on almost every aspect of the technique. The goal of this article is to provide a brief overview of factor analytic procedures to help school psychologists become more effective consumers of this type of research.

When is Factor Analysis Used?

In school psychology research, factor analysis is frequently used to evaluate the internal structure (i.e., factor structure) of psychoeducational tests (i.e., commercial ability measures, rating scales, etc.). For example, Dombrowski, McGill, and Canivez (2017) used factor analysis to evaluate the structure of the Woodcock-Johnson IV Tests of Cognitive Abilities (WJ IV) to determine the degree to which the instrument aligned with the theoretical structure posited by the test publisher. The results of these analyses are vital as they provide the statistical rationale for the standardized scores that are later computed for those measures and presented to clinicians as capable of being interpreted. Put simply, the resulting factor structure of an instrument is used to create the various index and composite scores that are interpreted by users. As a result, the *Standards for Educational and Psychological Testing* (American Educational Research Association [AERA], American Psychological Association [APA], & National Council on

Measurement in Education [NCME], 2014) encourage test publishers to provide this information in the technical manuals that accompany these tests and suggest that practitioners should interpret scores cautiously, if at all, that are not supported with appropriate factor analytic evidence. As "the ultimate responsibility for appropriate test use and interpretation lies predominantly with the test user" (AERA, APA, & NCME, 2014, p. 141), it is necessary for school psychologists to consider the degree to which factor analytic results support the interpretive procedures suggested in technical manuals and other related professional resources.

What is Factor Analysis?

There are two main types of factor analysis: exploratory factor analysis (EFA) and confirmatory factor analysis (CFA). Although both techniques are useful, they place different constraints on the researcher. EFA is less restrictive than CFA, requiring little input about the model prior to analyses. As such, it is regarded as a descriptive procedure that allows the data to “speak for itself” (Carroll, 1985, p. 26), and is typically utilized when a new instrument is being developed or when the theoretical structure for a dataset is not clear. Alternatively, in CFA, a researcher specifies a model *a priori* and then evaluates how well that model fits the data based upon resulting fit statistics. Thus, it is generally considered a preferred approach when the primary goal of research is to test theory. Whereas EFA analyses can be done in most omnibus statistical programs (SPSS, R, SAS), CFA typically requires the use of a specialized software program (EQS, Mplus, AMOS). In spite of these distinctions, both techniques are similar in that they require a researcher to make a number of subjective decisions which can complicate the replication of results.

Although EFA methods have been eclipsed by CFA methods in the school psychology literature over the last decade, they are considered to be complimentary procedures. That is, EFA typically precedes CFA and when the results from these procedures are in agreement, greater confidence can be placed in a structure for a test. It should be noted that CFA is a technique that falls under the broader umbrella of structural equation modeling (SEM), a topic that was recently reviewed in a previous article in this series (see von der Embse, 2016). Space limitations preclude a discussion of this important topic so the remainder of our discussion is focused more specifically on helping school psychologists to become more informed about EFA research.

Key Terms and Concepts

Observed Versus Latent Variables

As previously discussed, the fundamental goal of factor analysis is to disclose the latent structuring of variables. From this perspective it is important to make a distinction between observed and latent variables. Observed variables (also known as measured variables [MVs]) are data that are measured and available to a researcher (i.e., responses to rating scale items, cognitive subtest scores, etc.). In contrast, latent variables represent hypothetical psychological dimensions that are not directly observed but are inferred from the MVs. The resulting factors from an EFA are hypothesized to represent latent variables.

Common Factor Model

EFA adheres to the *common factor model* (see Figure 1 for several examples). From this perspective, a factor is a latent variable that accounts for the correlations between two or more MVs or indicators. Using a sample correlation or covariance matrix for a dataset, factor analysis partitions the variance of each indicator into two parts: (a) *common variance*, that reflects the variance that is shared between indicators due to a common cause; and (b) *uniqueness*, which is a combination of reliable variance that is unique to the indicator (also known as specificity) and error variance.

In Figure 1.1, Factor 1 (F1) is produced from the shared variance in the first and second MVs (MV 1-2) and is calculated from the correlation between those indicators. If this variance were removed from those variables, the correlation between those indicators would likely return to zero. Thus, in the common factor model, factors have direct effects (influence) on the MVs, as indicated by the direction of the path arrows. There is another important rule that deserves attention. That is that a factor requires a minimum of two indicators in order to be identified. As a result, a factor that is produced from only two indicators is referred to as a *just-identified* factor. Unfortunately, these types of factors and in some cases, whole models (i.e., all factors in the model are just-identified) are common in commercial ability measures despite the fact that it has long been suggested that factors should be composed of at least three to five variables in order to be measured well.

The data represented in Figure 1.1 results in no path between F1 and F2, which indicates that these factors are uncorrelated. A model with no path between the factors is referred to as an orthogonal factors model. However, this model is rarely tenable in school psychology due to the fact that many variables in psychology and education are correlated with each other. Alternatively, in an oblique factors model (Figure 1.2), the factors are allowed to correlate as indicated by the additional reciprocal path (arrows that are bidirectional) from F1 to F2. Further, moderate to strong factor correlations in an oblique factors model suggest that there is unexplained covariation and that it may be necessary to determine if an additional second-order factor may help explain these relationships. As illustrated in Figure 1.3, the specification of a second-order factor results in an additional layer being added to the model reflecting a more complex hierarchy thus, these models are frequently referred to as *hierarchical* models. In Figure 1.3, a second-order (general) factor is produced from the shared variance between F1 and F2 is calculated from the correlation between those factors. Whereas the first-order factors (F1 and F2) continue to exert a direct influence on the MVs, the influence of the general factor is mediated through F1 and F2. Thus, in a hierarchical model, the general factor has an indirect influence on the MVs.

It should also be noted that all of the models outlined in Figure 1 reflect a trait of a factorial model known as *simple structure*. That is all of the MVs are aligned with their theoretically proposed factors and load only on those factors. Although simple structure is desired by a researcher, it may not always reflect reality. For example, a MV may *cross-load* and align with more than one factor or the uniqueness terms for MVs may be correlated indicating that that there is some degree of shared variance between the indicators that is not accounted for by the factors. Although it is possible to model correlated terms in CFA, they are not permitted in EFA.

Factor Extraction Method

After determining that a dataset is suitable for factor analysis (i.e., adequate sample size, variables are normally distributed, intercorrelations are sufficiently large to indicate the presence of latent dimensions, etc.), one must determine a method for extracting the factors from the dataset (i.e., principal components [PCA], principle axis factoring [PAF], maximum likelihood [ML]). Whereas PAF is recommended, PCA is also commonly used in the school psychology literature due to the fact that it is the default extraction method in popular commercial software programs such as SPSS. However, PCA does not adhere to the common factor model and is not considered to be an appropriate factor analytic technique. As a result, practitioners should interpret EFA results produced from PCA with caution. Nevertheless, with adequate sample sizes, most methods will produce patterns and loadings and factor solutions that are generally equivalent to one another.

Number of Factors

Because competing models cannot be tested in EFA, the decision regarding how many factors to extract is critically important. This decision is not arbitrary as over-extraction can lead to the retention of spurious factors and under-extraction may result in an under-developed model. Although numerous rules of thumb have been proposed, the use of empirical tests (i.e., parallel analysis, minimum average partials) to aid decision making is considered best practice. As an example, Dombrowski et al. (2017) found that empirical criteria did not support the seven cognitive factors posited for the WJ IV by the test publisher. As a result, when a seven-factor model was forced to the data, it produced several factors that were not permissible (i.e., contained less than two salient subtest loadings per factor).

Analytic Rotation

After factors are extracted, a rotation is usually applied to the data in order to improve the interpretability of results. If the factors are assumed to be correlated, then an oblique rotation (e.g., promax, oblimin) should be employed. Whereas if the factors are assumed to measure traits that are not related in any meaningful way or the correlations between them are assumed to be trivial, then an orthogonal rotation (e.g., varimax) should be used. Rotational mathematics are quite complex however a simplified visual example of these rotational strategies is provided in Figure 2. The dotted lines represent the post-rotation axis. Notice that in the orthogonal rotation, the vectors for Factor 1 and Factor 2 maintain their 90º angle whereas in the oblique rotation the vectors converge, reducing the angle between them.

Interpreting EFA Results

Once these inputs are specified, a standardized solution reporting the loadings of all of the MVs on the factors, as well as *communality* and *uniqueness* estimates can be obtained (see Table 1 for an example for Figure 1.2). Communality estimates reflect the amount of variance in a MV that is accounted for by all of the factors combined whereas uniqueness estimates indicate the amount of variance in the MV that is not accounted by the common factors. Both estimates can range from zero to one (note that both terms when added together sum to one). At this point, it is important to inspect the pattern and strength of the loadings for theoretical consistency. Loadings that are ≥ .30 are considered to be salient and indicate that the MV aligns relatively well with that factor. As previously noted, a factor should be produced from two or more salient loadings. One can obtain the amount of variance in a MV that is accounted for by a common factor by squaring the factor loading. For example, in Table 1, F1 accounts for 67% of the reliable variance in MV1.

If the obtained solution is determined to be adequate, the researcher then has to name the factors. This requires a subjective decision based upon the alignment of the MVs with the factors. Interpretation is complicated by the fact that MVs are permitted to load on all of the factors in EFA (see Table 1). As an example, let's assume that MV1 and MV2 are both measures of visual ability. Since they both load on the same common factor (F1), a logical decision would be to interpret that factor as reflecting a broad visual processing construct. However, it is important to note that just because a factor can be extracted does not mean that it reflects a legitimate psychological dimension. For instance, if MV1 and MV2 measured different cognitive processes, then the naming of F1 becomes more difficult. As a result, school psychologists should be mindful of the *naming fallacy*—the false belief that the name of a variable accurately reflects what that variable measures or that it even measures a legitimate construct at all.

Guidelines for Evaluating EFA Findings

Contrary to widespread misconceptions, EFA should not be a blind process in which "all manner of variables or items are thrown into a factor-analytic 'grinder' in the expectation that something meaningful will emerge" (Pedhazur & Schmelkin, 1991, p. 591). After all, almost anything can be uncovered if one is willing to engage in a visionless fishing expedition (i.e., extracting different numbers of factors, using different methods and rotation strategies). Given the amount of subjectivity that is involved in the EFA process, it is incumbent upon the researcher to provide a substantive justification for each of these decisions. Relatedly, when evaluating any factor analytic study, it is important to consider the degree to which the analyses are consistent with the structure that is most likely for the data. For instance, it is common for researchers to validate intelligence tests via EFA using a correlated (oblique) factors approach when the scores that are provided to clinicians for that instrument imply a hierarchical model (e.g., FSIQ). It is possible to conduct a hierarchical EFA in most statistical programs however these analyses require an additional analytical step that is beyond the scope of the present discussion (see Dombrowski et al. for an example).

It is also important to inspect the pattern of loadings to ensure that they are theoretically consistent and free of weak and/or problematic loadings such as cross-loading. As previously mentioned, cross-loading implies that two or more factors influence a variable, presenting an interpretive confound for the clinician. A pattern of weak loadings is also problematic as it indicates that a factor may not be measured well within a dataset.

Finally, school psychologists should consider the degree to which the results are consistent with previous factor analytic research (both EFA and CFA). As stated by Gorsuch (2003), "the ultimate arbiter in science is well established replication" (p. 153). As EFA and

CFA provide answers to different empirical questions, contradictory results are commonly reported within the school psychology literature.

Conclusion

EFA is a versatile technology that has been useful for numerous advances in applied psychology. For example, Carroll (1993) relied exclusively upon this technique to develop his three-stratum model of cognitive abilities. Despite its strengths, it is not without flaws. Commonly utilized EFA techniques and available statistical programs do not provide a robust test for competing models for a dataset. Nevertheless, it remains popular within school psychology research because of its ability to inform the more complex modeling that is required in CFA. Thus, practitioners should have a working understanding of the strengths and limitations of this technique in order to effectively use the results produced from EFA or factor analytic studies in general to support individual decision-making in clinical practice.

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For More Information

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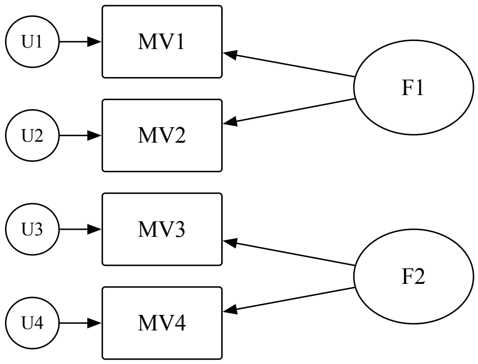
Table 1

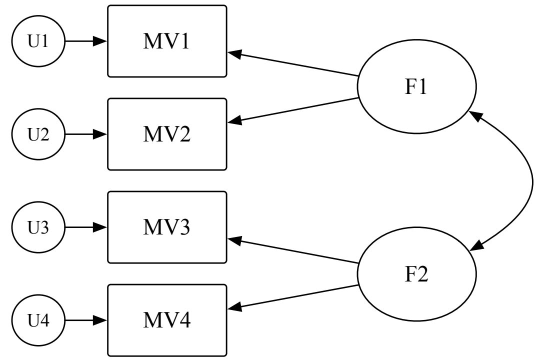
Sample Exploratory Factor Analysis Results with Oblique Pattern Coefficients

	Oblique Solution			
Variable	Factor I	Factor II	h^2	u^2
Measured Variable 1	**.82**	.05	.76	.24
Measured Variable 2	**.80**	.08	.77	.23
Measured Variable 3	-.01	**.75**	.58	.42
Measured Variable 4	.05	**.72**	.59	.41

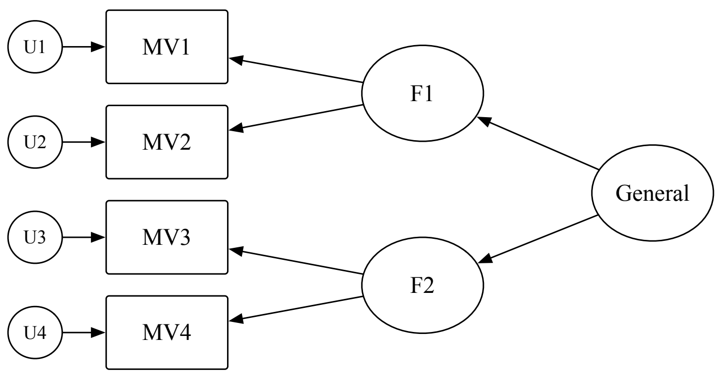
Note. Salient loadings ≥ .30 are denoted in bold. h^2 = communality; u^2= uniqueness.



1.1 Orthogonal Factors Model



1.2 Correlated (Oblique) Factors Model



1.3 Hierarchical Model

Figure 1. Graphical display of the common factor model for different structural representations of an example featuring two common factors and four measured variables. U = uniqueness, MV = measured variable, F = factor.

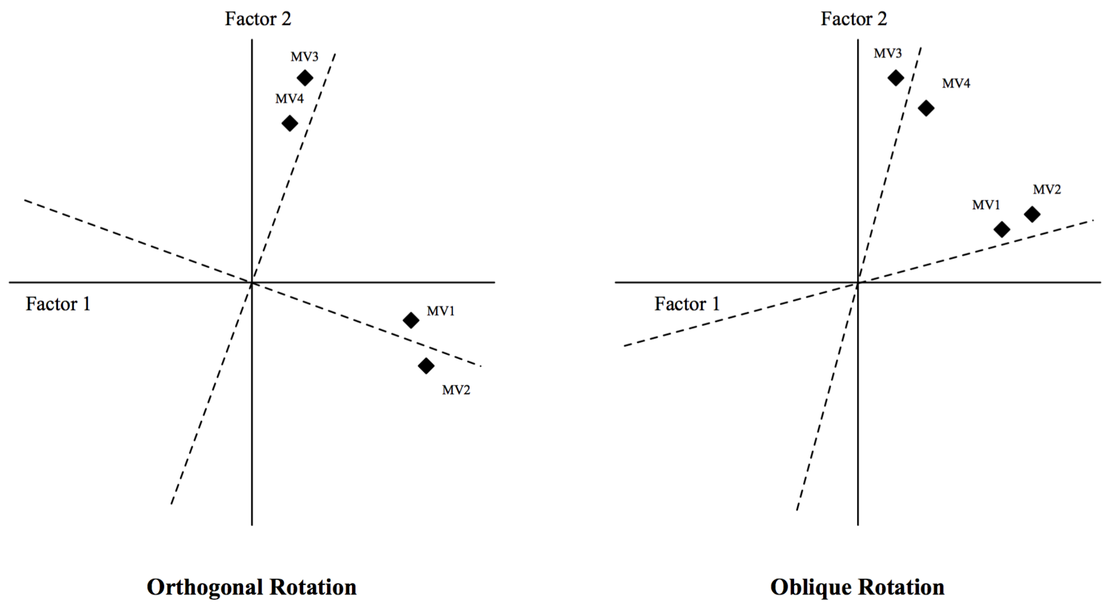


Figure 2. Example of an analytic rotation in exploratory factor analysis.